COMBINATORICS - COUNT 'EM UP

MATHCAMP 2K

(please ignore number agreement with pronouns. Jane Austen used "they" as a third person singular pronoun, so I feel justified to do so as well.)

1. Here are questions about how many words one can make:
   (a) How many words can one make with the letters of the words "statistics", "counterintelligence", "transubstantiation"?
   (b) How many five-letter words can you make with those words?
   (c) How many seven-letter words can you make with those words?
   (d) How many nine-letter words can you make with those words?

2. Everyone puts their name on a slip of paper, the slips are mixed up, and everyone is handed a random slip. Since the purpose of this little game is for people to meet somebody else, nobody wants to get their own name back. In the following, try to calculate the number of ways for some small group of people, say 6, and then do it for a general n number of people.
   (a) How many different ways can the slips of paper be handed out?
   (b) How many different ways can exactly one person get back their own name?
   (c) How many different ways can exactly two people get back their own name (three, four,five, ...)?
   (d) How many different ways can everyone get a different person's name?

3. Suppose you have a bag of N real numbers written on slips of paper. You're going to pull the slips out one by one. Try these problems for $N = 6$ and then general N.
   (a) How many different ways can you arrange the numbers so that the largest number is last?
   (b) How many different ways can you arrange the numbers so that the largest number is an even position?
   (c) How many different ways can you arrange the numbers so that the largest number comes before the second largest number?

4. My mother just \$1300 at slots in Vegas (it's true!) Let's play a game to model the slots: I take a bag full of symbols, out of which I pull three at random (without replacing them). Inside the bag are 3 bells, 5 cherries, 20 moons, and 50 stars.
   (a) For the big jackpot, one must pick out 3 bells. What's the probability of this happening?
   (b) For one type of jackpot, one has to get exactly 2 moons. What's the probability of this occurring?
   (c) What's the probability of pulling out exactly one bell and at least one cherry?

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