COMBINATORICS 2 – ELECTRIC BUGALOO

MATHCAMP '00

Some Short and Sweet Problems

1. There's a game in *Games Magazine* called Word Rummy. What they have is a grid that's 4×13 filled with letters. As you may guess, each grid square is associated with a normal card (and some letters are repeated throughout the grid). One has a word rummy if one has a rummy hand, which is formed by two groups: 3 or 4 of a kind, and 3 or 4 consecutively numbered cards of the same suit to make up a total of 7 cards. Order matters, because you're trying to see if you have a word. For example, one may have something that looks like this: MEATPOD (let's pretend it's a word) formed from the cards $A\spadesuit$ $A\heartsuit$ $A\diamondsuit$ $7\heartsuit$ $8\heartsuit$ $9\heartsuit$ $10\heartsuit$. However, one is *not* allowed to form METAPOD because then the group of aces has been mixed with the consecutive card group. However, one would be allowed to use EMATPOD, for the aces can be taken in any order. So the obvious question is, given that I'm bad at word puzzles and want to try all possibilities, how many word rummys are there?
2. So we want to organize the swing class, making as many mixed-gender couples as possible. How many different ways can we make dancing pairs if:
   (a) there are 15 women and 15 men?
   (b) there are 13 women and 17 men?
   (c) there are 2 women and 28 men?
3. How many non-negative integer solutions are there to the following equations/inequalities?
   (a) $x + y + z = 8$
   (b) $x + y + z + w = 18$
   (c) $0 \leq x_1 + x_2 + x_3 \leq 10$
4. How many selections of 3 numbers can be made from $\{1, 2, \ldots, 99, 100\}$ if no two consecutive numbers can be included?

Poker time!

You're thinking about playing poker with Meep the knife Friday evening (10:30 pm, big ole common room), but you want to check out some probabilities first. We've figured out all the probabilities for 5-card stud, where you get dealt 5 cards and that's it, but she's likely to be playing 7-card stud and 5-card draw.

5-card draw. In 5-card draw, you are dealt 5 cards, then you can choose to throw some away and get new ones dealt to you. To keep it easy, let's consider a common case: you've been dealt a pair, and you are going to throw away the three cards not in the pair to see if you can improve the hand. Figure out the probabilities of:

1. Ending up with 4-of-a-kind
2. Ending up with a full house
3. Ending up with 3-of-a-kind
4. Ending up with one pair (that is, nothing improved)

How likely is it that your hand will improve on exchange?

7-card stud. In 7-card stud, you're dealt 7 cards and you don't get to exchange any of the cards. The value of your hand is equal to the best 5-card poker hand you can create from your 7 cards. Try figuring out the probabilities for the different kinds of poker hands again: royal flush, four-of-a-kind, straight flush, full house, flush, straight, three-of-a-kind, two pair, one pair, and high card (a.k.a. nothing). Warning: you must be careful, for perhaps with 7 cards you can get two kinds of hands at the same time.