# DISCRETE MATH – PROBABILITY

MATHCAMP '00

(please ignore number agreement with pronouns. Jane Austen used "they" as a third person singular pronoun, so I feel justified to do so as well.)

## 1. ESP experiment

The Meep Psychic Community$^{TM}$ is trying to make their hiring process a little more quantified, so they're administrating the following test to prospective "friends": a card with one of the 5 mystic symbols (representing earth, wind, fire, water, and money) is picked at random for each of 30 trials. People are supposed to write down which symbol was picked, using whatever supernatural powers they can draw on.

1. What is the probability that someone guessing at random gets all 30 correct?
2. What is the probability that they would get none correct?
3. How many trials would you expect a person to get right if they were guessing at random?
4. What's the probability that someone gets more than that expected value right?
5. Suppose MPC$^{TM}$ wanted to score this test so that someone simply guessing should on average score of zero. How could this be done?

## 2. Pokemon League Tournaments (Gotta Catch 'Em All)

1. In the Pokemon League, there are 150 official Pokemon one can choose among. In the first level tournaments, the trainers are allowed only one Pokemon for the match. Assuming that each Pokemon is equally likely, what is the greatest number of Pokemon trainers that can participate in the tournament so that the probability of any of them having the same Pokemon is less than 50%?
2. Now, in the next level tournament, one uses two Pokemon in the match. Now what is the greatest number of trainers so that the probability that any trainer has *both* the same Pokemon as another trainer is under 50%? What if we want the probability of any of the two Pokemon matching between trainers to be less than 50%?
3. Now, in the highest level tournament, one can use *three* (count 'em – three!) Pokemon. I'm tired of typing, so figure out what relevant questions I would ask (considering the questions above), and then answer them.

## 3. Roll Those Dice and Flip Those Coins

1. You roll a die until you get a 6. How many times do you expect roll the die? What is the expected total of all throws before you throw a 6? (e.g., you throw 1, 3, 5, 6 for a total of 4 rolls and a sum of 9 (without the 6))
2. Meep and Helen decide to bet on the rolls of 2 normal dice. Meep bets that a 12 will be rolled first, Helen bets that two 7s will be rolled consecutively first. They keep rolling until one wins. What's the probability that Meep will win? What is the expected number of dice rolls in this game?

*Date*: July 2000.

3. Pepys evidently liked playing dice alot, and liked to make certain bets on dice. However, he couldn't figure out which were the best bets to make and so he wrote Newton about the following three bets: a bet that the dice-roller will get at least 1 six when six dice are rolled; that they will get at least 2 sixes when 12 dice are rolled; or that they will get at least 3 sixes when 18 dice are rolled. Which of these three events is most likely to occur?
4. Suppose you repeatedly toss a fair coin until you get 2 heads in a row. What is the probability that you stop on the 10th toss? What if you stop when you get two of the same results in a row (like HH *or* TT)?
5. Two coins are given. One is fair (prob. of heads = $\frac{1}{2}$) and one is biased (prob. of tails = $\frac{2}{3}$). One coin is tossed once and results in heads. The other coin is tossed 3 times resulting in 2 heads. Which coin is more likely to have been the biased one?

## 4. It Depends – Grab Bag of Questions

1. A drawer contains red socks and black socks. When two socks are drawn at random, the probability that both are red is $\frac{1}{2}$. How small can the number of socks in the drawer be? How small if the number of black socks is even?
2. You're going treasure hunting near Barbados, but you are doing so illegally. Therefore, you have to get in and get your treasure quickly. You know this sunken cruise ship contains two safes, one which has two gold bars and one silver bar and the other which contains 101 gold bars and 100 silver bars. However, you cannot tell the safes apart. So your plan is the following: get into the ship, cut a hole in one of the safes (you only have time to cut a small hole, so you can't see inside) and pull out two bars. Now you have a choice: after you pull out the first bar, you can put it back in before you pull out a second. Or you can just keep it out. Which of these two methods is best for determining which safe has the jackpot?
3. In a country where girls are favored over boys, each family keeps having children until a boy is born (so as to keep the number of boys down). If the probability of each birth being of a particular gender is 50-50, after a generation of this strategy, what will be the proportion of females to males?
4. Karl asks his dad for more pocket money, but his dad suggests the following: "You'll get more if you play 3 games of chess with your mom and me, and you win two consecutive games. Your mom and I will take turns playing, and you may choose whom to play first." If Karl's mom is a better player than his dad, whom should Karl play first to have a better chance?
5. Of a population, two-thirds are vaccinated against a certain disease. Of all the vaccinated people, one-fifteenth are infected and of all the infected people, six-sevenths are not vaccinated. Is the vaccine preventing disease? How likely is one to get the dissease, if one is not vaccinated?
6. You have a cloth bag with one marble inside (it could be either black or white). You add a white marble, shake the bag around, and pull one of the two marbles out. It is white. What is the probability that the remaining marble is white?
7. Suppose you can choose to enter one of two lotteries, both of which have the same jackpot. In the first lottery, you have to pick 6 out of 50 numbers correctly. In the second lottery, you have to pick 5 out of 55 numbers correctly. Which is the better lottery to enter?
8. The keeper of the web page 'The premature death of rockstars' (www.av1611.org/rockdead.htm) argues that rock stars do not live as long as the general population. He states that the average age at death for rock stars is 36.9 years and it's 75.8 years for the general population. I can think of at least two things wrong with his analysis (you don't even have to look at the page to consider why these figures may be misleading).