PROBABILITY INFO

MATHCAMP '00

1. Definitions

Here are some definitions used in set theory and probability. In all the following, the example I shall use is that of flipping a coin 4 times.

- S is our *probability space*, a set whose elements are called *outcomes*. I will use lower case letters to indicate outcomes. For example, an outcome for my coin flipping would be heads-tails-tails-tails. The probability space would be the set of all 16 possible outcomes.
- *Events* are subsets of S. I use upper case letters to indicate events. Example: getting at least 1 head is an event (let's call it A).
- I will call our *probability measure* P, which is a real-valued function on subsets of S. So we say that $P(A)$ is the probability of event A occurring. Example: the probability of the above event, P(A) = $\frac{15}{16}$.
- A^C is called *A complement.* $A^C = S \setminus A = \{x : x \in S, x \notin A\}$. In words, A complement is the set of every outcome in the probability space that is not in event A. Example: the complement of the above event A is the event that one gets no heads in the 4 flips.
- $A \cup B$ is called *A union B* or the union of A and B. $A \cup B = \{x : x \in A, \text{ or } x \in B\}$. So the union of A and B are all the outcomes that are in either event or both.
- $A \cap B$ is called *A intersect B* or the intersection of A and B. So this is like something *or* something else will happen. $A \cap B = \{x : x \in A, \text{ and } x \in B\}$. So the intersection of A and B contains only those outcomes which are in both events. This is like saying that something *and* something else will happen.
- Sometimes two events have no common outcomes, or $A \cap B = \emptyset$. In that case we say A and B are *mutually exclusive*. Example: the event that one gets all heads and the event that one gets 2 tails and 2 heads are mutually exclusive, for they can't happen at the same time.
- There's a special measure using P called *conditional probability*; it's a real-valued function that takes two events as its input. It's notated as $P(A|B)$ which is read as "the probability of A given B". It's defined as $P(A|B) = \frac{P(A \cap B)}{P(B)}$. If $P(A|B) = P(A)$, then we say A and B are *independent* events. Example: Let A = getting all heads, B = getting heads on first coin toss, C = getting heads on second coin toss. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{16}}{\frac{1}{2}} = \frac{1}{8} \neq P(A)$. So A and B aren't independent events. However, if you figure it out, B and C *are* independent events - they have no effect on the probability of each other. (In the case of coins, it's because the coin has no 'memory' which affects its tosses.)

2. The Postulates

The following three postulates characterize all probability measures.

- $P(S) = 1$. This rule means that the measure of the entire space is one – probability of some outcome occurring is one.
- $P(A) \geq 0, \forall A \subset S$. This rule means there's no such thing as negative probability for any event.
- If A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B)$.

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3. The Rules

All of the following can be derived from the Postulates and Definitions of probability. You can try to derive these as an exercise.

- $P(A) \leq 1, \forall A \subset S$
- $P(A) = 1 - P(A^C)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(\emptyset) = 0$
- $P(A \cap B) = P(A|B)P(B)$
- If $P(A|B) = P(A)$, then $P(B|A) = P(B)$.
- If A and B are independent events then $P(A \cap B) = P(A)P(B)$.
- $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$